Gravitational Wave Background Radiation from Supermassive Black Hole Binaries on *Eccentric Orbits*

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§ 1.Introduction

An ensemble of gravitational waves (GWs) from a number of inspiraling binaries of compact objects at different redshift

=> Stochastic gravitational wave background radiation (GWBR)

SMBH-SMBH : $f \sim 10^{-9} - 10^{-6}$ Hz

(e.g. Wyithe & Loeb 2003, Enoki et al. 2004, Sesana et al. 2004,)

WD-WD/WD-NS : $f \sim 10^{-5} - 10^{-1}$ Hz

(e.g. Farmer & Phinney 2003)

The GWBR from supermassive black hole (SMBH) binaries can be detected by pulsar timing measurements.

=>e.g. The Parkes Pulsar Timing Array (PPTA) project

*Eccentricity ?

In previous studies of GWBR from SMBH binaries, it is assumed that all binaries are on circular orbits.

However, binary orbits are generally eccentric.

- evolving SMBH binary due to dynamical friction (e.g. Fukushige et al. 1992, Zier 2006)
- SMBH - IMBH (Matsubayashi et al. 2005)
- SMBH binary + SMBH (Iwasawa et al. 2006)
- SMBH binary + gas disk (e.g. Armitage & Natarajan 2005)

An eccentric binary emits GWs at all integer harmonics of the orbital frequency.

⇓

The spectral energy distribution (SED) , the power and the timescale of GW radiation are different from those of a binary on a circular orbit, even if masses and semi-major axis of both binaries are the same.

In this study,

Semi-analytic model of
galaxy formation + SMBH formation (SA-model)

↓

(Nagashima et al 2001,
Enoki et al 2003)

SMBH coalescence rate

+

GW SED from a eccentric binary

↓

GWBR spectrum from SMBH binaries
on ***Eccentric Orbits***

§ 2.Gravitational Wave from a Binary (Peters & Mathews 1963)

The total GW luminosity of a Keplerian binary with orbital frequency f_p and orbital eccentricity e :

$$L_{\rm GW}(M_1, M_2, f_p, e) = L_{\rm GW,circ}(M_1, M_2, f_p) F(e),$$

where

$$L_{\rm GW,circ}(M_1, M_2, f_p) = \frac{32}{5}\frac{G^{7/3}}{c^5} M_{\rm chirp}^{10/3} (2\pi f_p)^{10/3}$$
$$= 4.7 \times 10^{48} \left(\frac{M_{\rm chirp}}{10^8\ M_\odot}\right)^{10/3} \left(\frac{2 f_p}{10^{-7}\ {\rm Hz}}\right)^{10/3} {\rm erg}$$

$$M_{\rm chirp} \equiv [M_1 M_2 (M_1 + M_2)^{-1/3}]^{3/5}$$

The total GW luminosity of a Keplerian binary on a circular orbit.

and

$$F(e) \equiv \frac{1 + 73e^2/24 + 37e^4/96}{(1 - e^2)^{7/2}}$$

*GW SED from a binary on eccentric orbit

The total power of GW emission is distributed into each the power of the *n*th harmonic of the orbital frequency, $L_{\mathrm{GW,circ}}(M_1, M_2, f_p)\, g(n,e)$, with rest-frame GW frequency, $f_r = nf_p$.

$g(n,e)$ is GW frequency distribution function expressed as

$$g(n,e) \equiv \frac{n^4}{32} \left\{ \left[J_{n-2}(ne) - 2eJ_{n-1}(ne) + \frac{2}{n} J_n(ne) + 2eJ_{n+1}(ne) - J_{n+2}(ne) \right]^2 + (1-e^2)\left[J_{n-2}(ne) - 2eJ_n(ne) + J_{n+2}(ne)\right]^2 + \frac{4}{3n^2}\left[J_n(ne)\right]^2 \right\} \quad (2$$

where J_n is the *n*th order Bessel function.

It can be shown that

$$\sum_{n=1}^{\infty} g(n,e) = F(e).$$

The SED of GW:

$$L_{f_r}(e, t_p) = L_{\mathrm{GW,circ}}(f_p) \sum_{n=1}^{\infty} g(n,e)\,\delta(f_r - nf_p).$$

*The timescale of GW from an eccentric binary

The timescale emitting of GW: $$\tau_{\rm GW} \equiv f_p \frac{dt_p}{df_p}.$$

The timescale emitting GW of a binary on eccentric orbit:

$$\tau_{\rm GW}(M_1, M_2, f_p, e) = \frac{\tau_{\rm GW,circ}(M_1, M_2, f_p)}{F(e)}$$

where

$$\begin{aligned}\tau_{\rm GW,circ}(M_1, M_2, f_p) &= \frac{5}{96}\left(\frac{c^3}{GM_{\rm chirp}}\right)^{5/3}(2\pi f_p)^{-8/3} \\ &= 1.2\times 10^4\left(\frac{M_{\rm chirp}}{10^8\ M_\odot}\right)^{-5/3}\left(\frac{2f_p}{10^{-7}\ {\rm Hz}}\right)^{-8/3}{\rm yr}.\end{aligned}$$

The timescale emitting GW of a binary on a circular orbit

§ 3. GWBG from binaries on eccentric orbits

The present day GWBR energy density:

$$\rho_{\rm GW} c^2 = \int_0^\infty \int_0^\infty n_c(z) \frac{1}{1+z} \frac{dE_{\rm GW}}{df_r} df_r\, dz = \int_0^\infty \int_0^\infty n_c(z) \frac{1}{1+z} f_r \frac{dE_{\rm GW}}{df_r}\, dz\, \frac{df}{f}\,.$$

$n(z)$: The number density of GW sources
$E_{\rm GW}$: the energy emitted in GW from a source
$f_{\rm r}$:the GW frequency in the rest frame
f : the observed GW frequency

$$\rho_{\rm GW} c^2 \equiv \int_0^\infty \frac{\pi}{4} \frac{c^2}{G} f^2 h_c^2(f) \frac{df}{f}\,,$$

The characteristic amplitude of GWBG spectrum (Phinney 2001):

$$h_c^2(f) = \frac{4G}{\pi c^2 f} \int_0^\infty n_c(z) \left(\frac{dE_{\rm GW}}{df_r} \right)\Bigg|_{f_r = f(1+z)} dz\,.$$

For binaries

$$h_c^2(f) = \frac{4G}{\pi c^2 f} \int dM_1 dM_2 dz\, n_c(M_1, M_2, z) \left(\frac{dE_{\rm GW}(M_1, M_2)}{df_r} \right)\Bigg|_{f_r = f(1+z)}$$

$n(M_1,M_2,z)$: The number density of binaries

*The power spectrum of GWBG from binaries on eccentric orbits

$$h_c^2(f) = \frac{4G}{\pi c^2 f} \int dM_1 dM_2 dz\, n_c(M_1, M_2, z) \left(\frac{dE_{\rm GW}(M_1, M_2)}{df_r} \right) \Big|_{f_r = f(1+z)}$$

$$\frac{dE_{\rm GW}}{df_r} = \int_0^{t_{\rm life}} L_{f_r}(t_p) dt_p.$$

The SED of GW:

$$L_{f_r}(e, t_p) = L_{\rm GW,circ}(f_p) \sum_{n=1}^{\infty} g(n, e) \delta(f_r - n f_p).$$

The power spectrum of GWBR from binaries on circular orbits:

$$h_c^2(f) = \frac{4\pi c^3}{3} \int dM_1 dM_2 dz\, n(M_1, M_2, z)(1+z)^{-1/3} \left(\frac{GM_{\rm chirp}}{c^3} \right)^{5/3} (\pi f)^{-4/3} \times \sum_{n=1}^{\infty} \left(\frac{2}{n} \right)^{2/3} \frac{g(n,e)}{F(e)}$$

Note that the eccentricity, e, is a function of the observed GW frequency f :

$$e = e(f_p / f_{p,0}, e_0) = e(f_r / n f_{p,0}, e_0) = e[f(1+z)/n f_{p,0}, e_0]$$

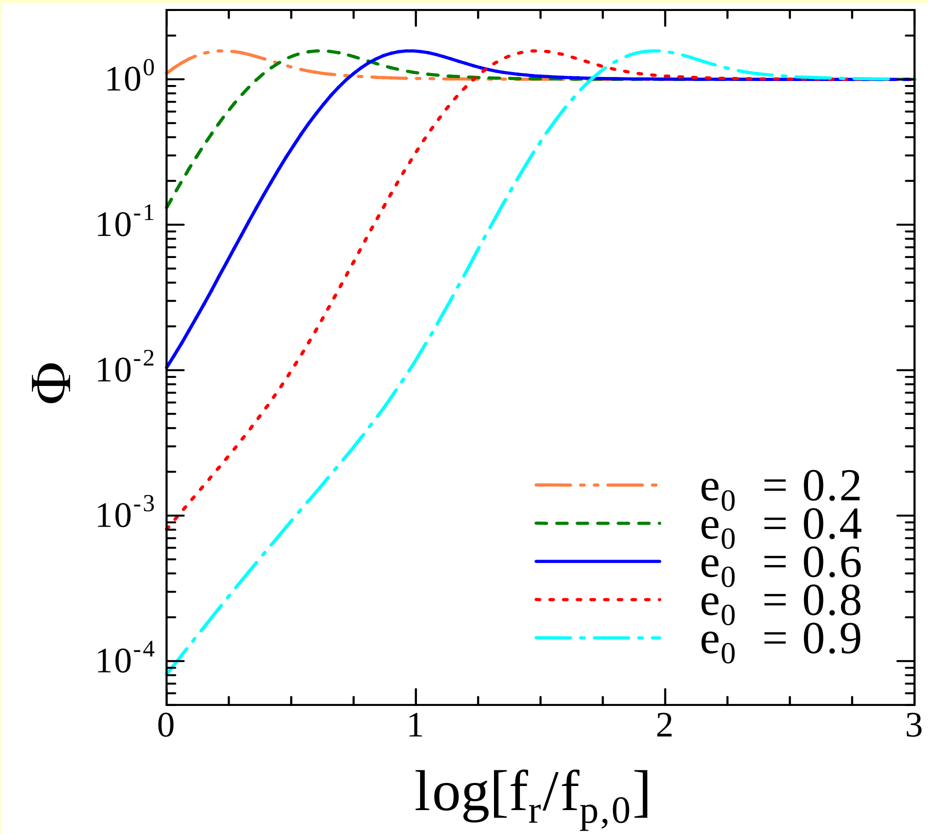
$$\frac{f_p}{f_{p,0}} = \left\{ \frac{1 - e_0^2}{1 - e^2} \left(\frac{e}{e_0} \right)^{\frac{12}{19}} \left[\frac{1 + \frac{121}{304} e^2}{1 + \frac{121}{304} e_0^2} \right]^{\frac{870}{2299}} \right\}^{-3/2}$$

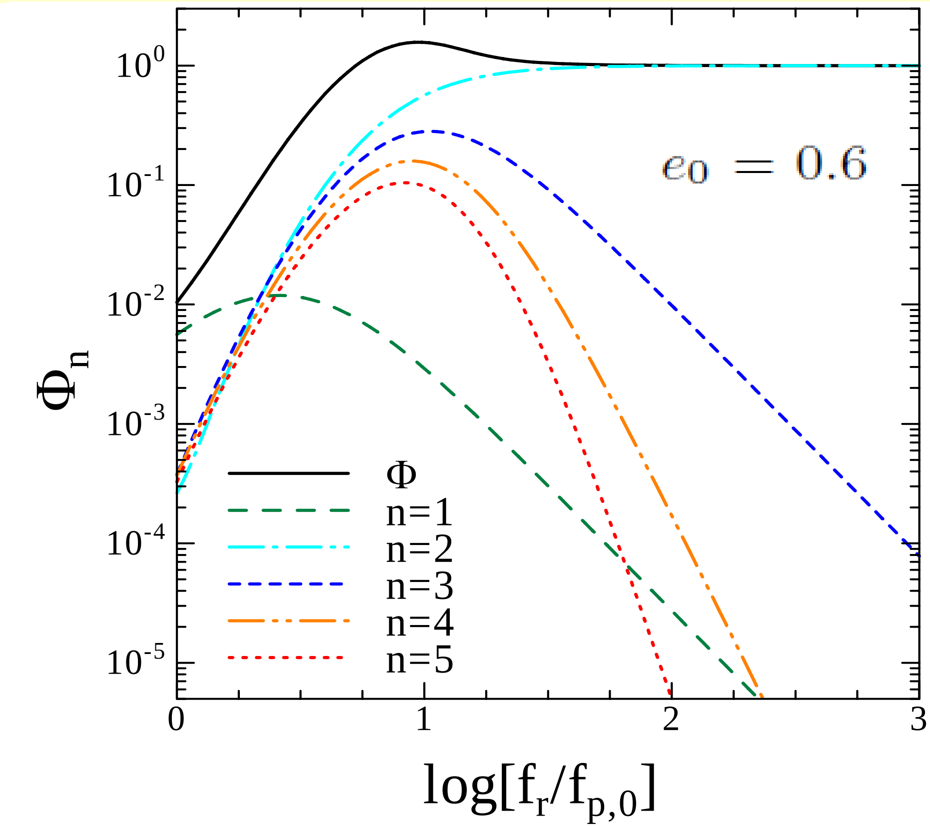
*Effects of eccentricity on GWBG

The strength of the effect of eccentricity :

$$\Phi \equiv \sum_{n=1}^{\infty} \Phi_n,$$

$$\Phi_n \equiv \left(\frac{2}{n}\right)^{2/3} \frac{g(n,e)}{F(e)}.$$

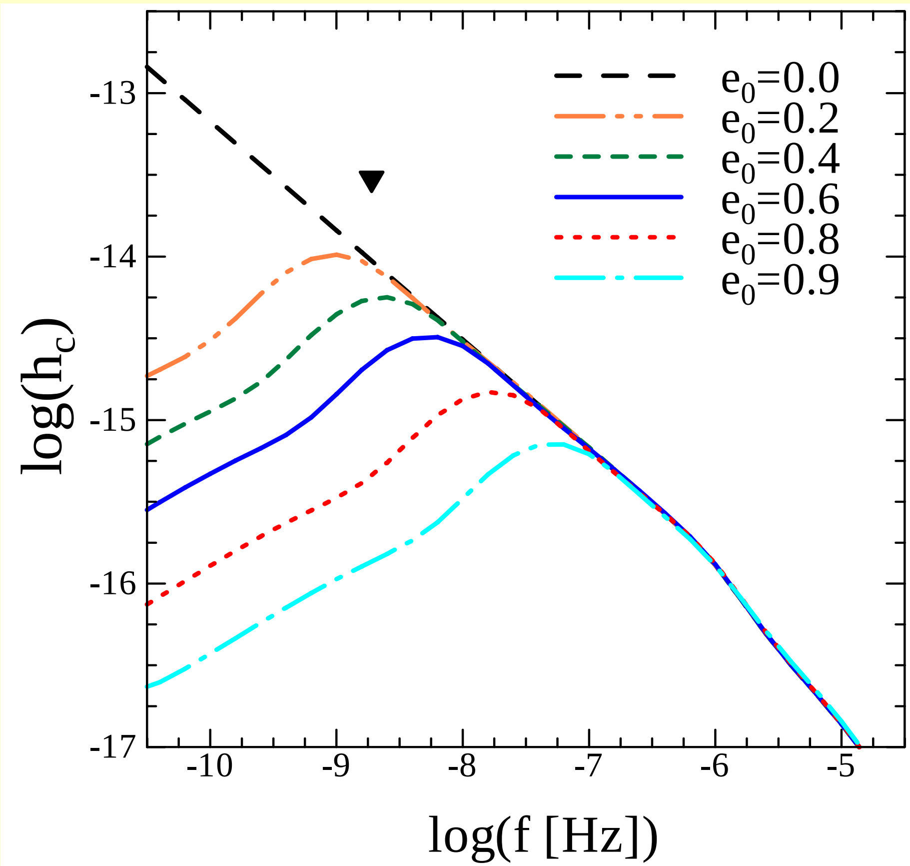




The power spectrum is suppressed at lower frequencies and is amplified at intermediate frequencies.

§ 4. Power spectrum of the GWBR from SMBH binaries

To estimate the number density of coalescing SMBH binaries, we use a semi-analytic model of galaxy + SMBH formation (Enoki et al. 2003)



▼ : the current limit from pulsar timing measurements.
(Lommen 2002)

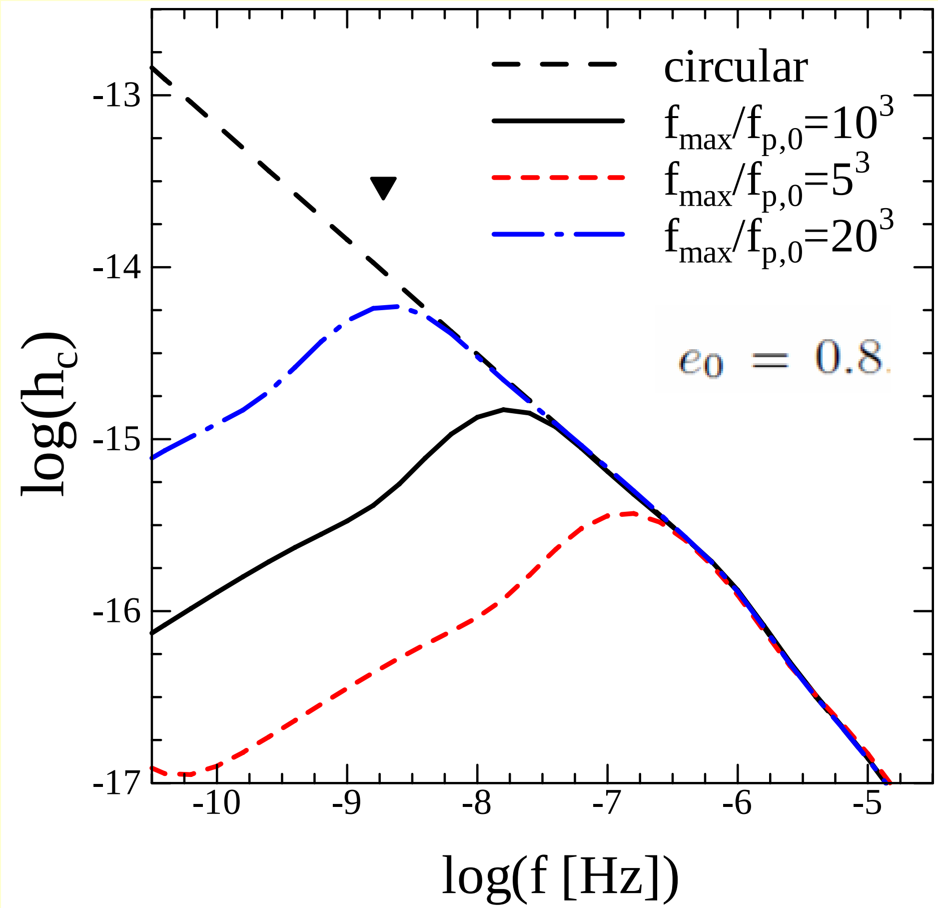
The initial eccentricities, e_0, are given at $f_p/f_{p,\,max} = 10^{-3}$ (a = $300R_S$).

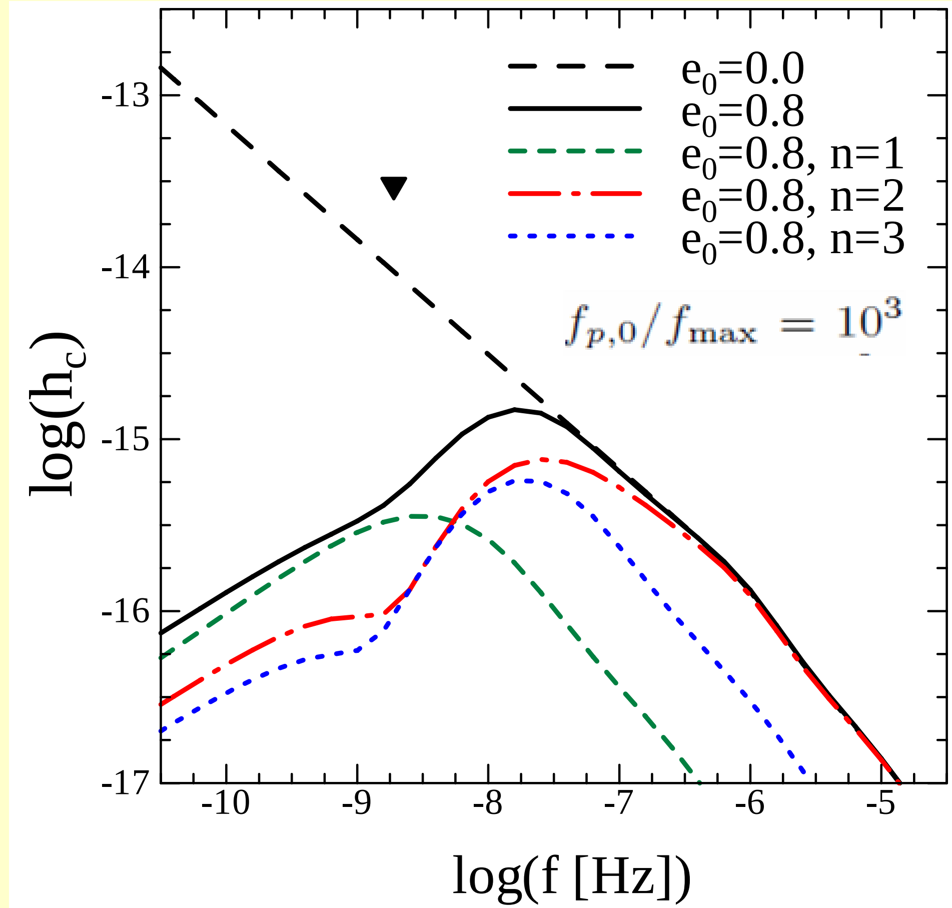
Power spectra at $f \sim$ 1n – 1mHz are suppressed due to the effect of eccentricity.

At $f \sim 1\ \mu$ Hz, the spectrum changes its slope owing to lack of power associated with the upper limit frequency:

$f_{p,max} \sim 5 \times 10^{-5}\ (M_{BH}/10^8 M_{sun})^{-1}$ Hz
[$3\times R_S$ (Schwarzschild radius)]

*GWBG spectrum from eccentric SMBH binaries





$$a^3 = GM_{\rm tot}/(2\pi f_p)^2$$

$$a = 0.5\times10^{-2}\times(M/10^8 M_{sun})^{1/3}(f_p/10^{-8}\,\mathrm{Hz})^{-2/3}\,\mathrm{pc}$$

*Effects of galaxy formation processes on SMBHs

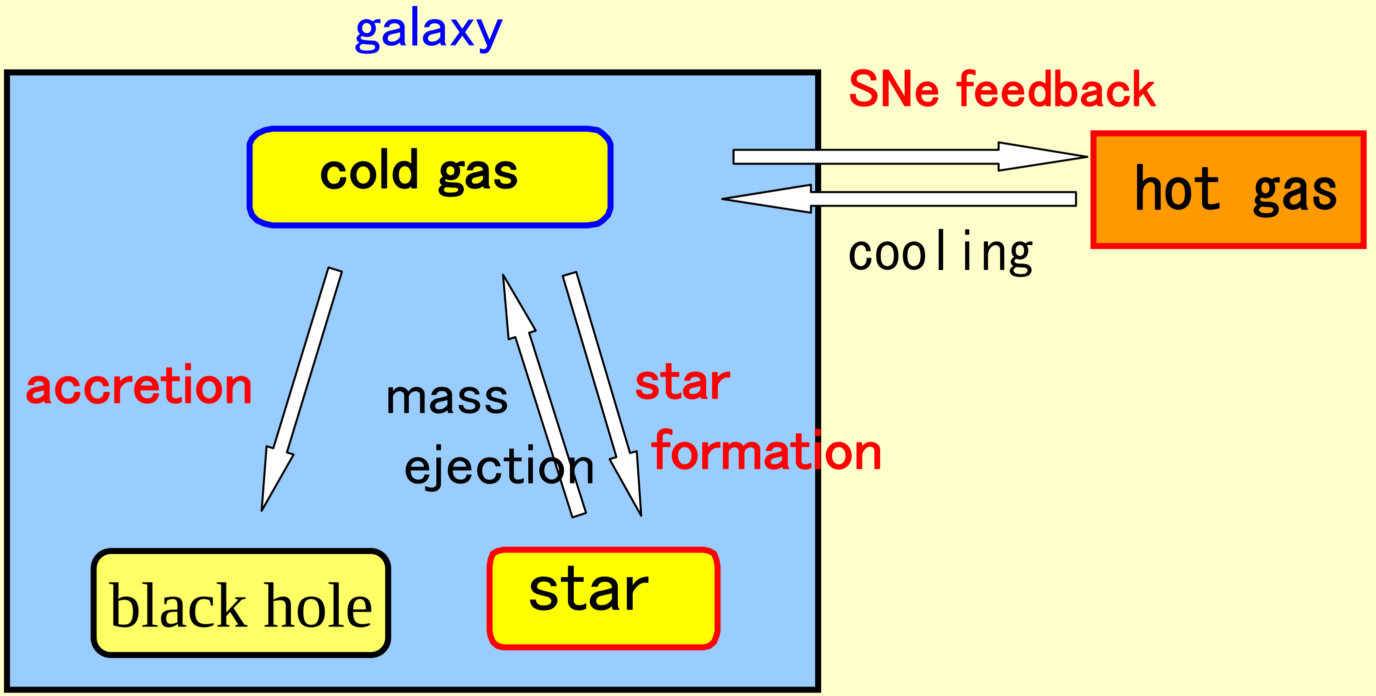
The dominant mass growth process of SMBHs are
the accretion of cold gas, which is also the material for stars.

*Star Formation

cold gas => star

*SNe feedback

cold gas => hot gas

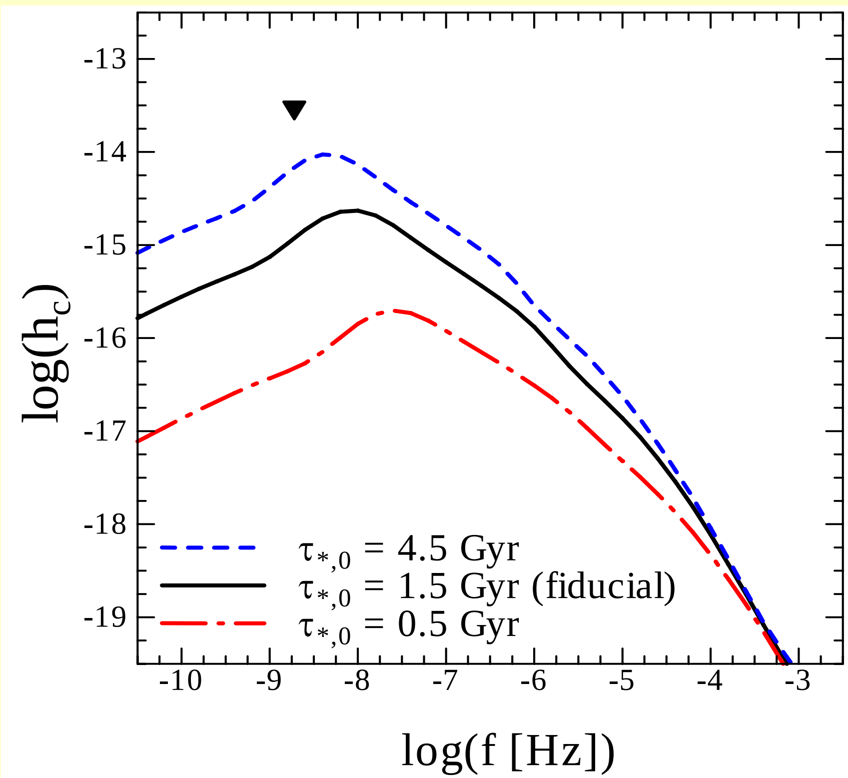


*Effects of galaxy formation processes on the GWBR

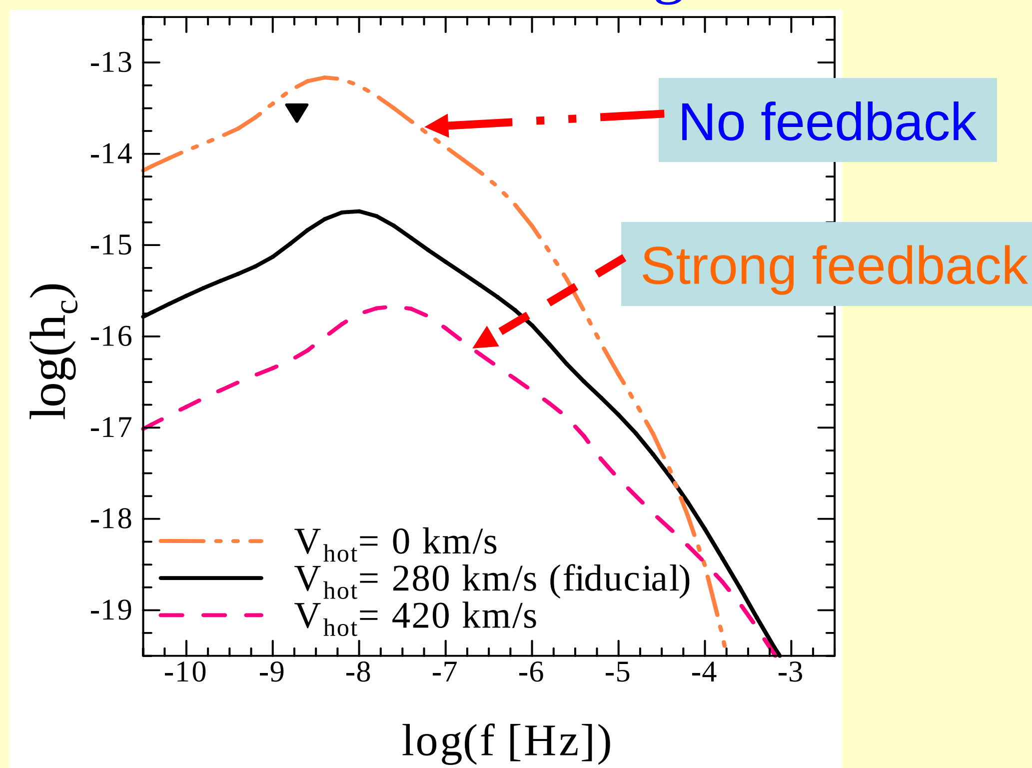
The dominant mass growth process of SMBHs are
the accretion of cold gas, which is also the material for stars.

=> Star formation & SNe feedback affect the power spectrum of the GWBR from SMBH binaries.

Star formation time-scale:

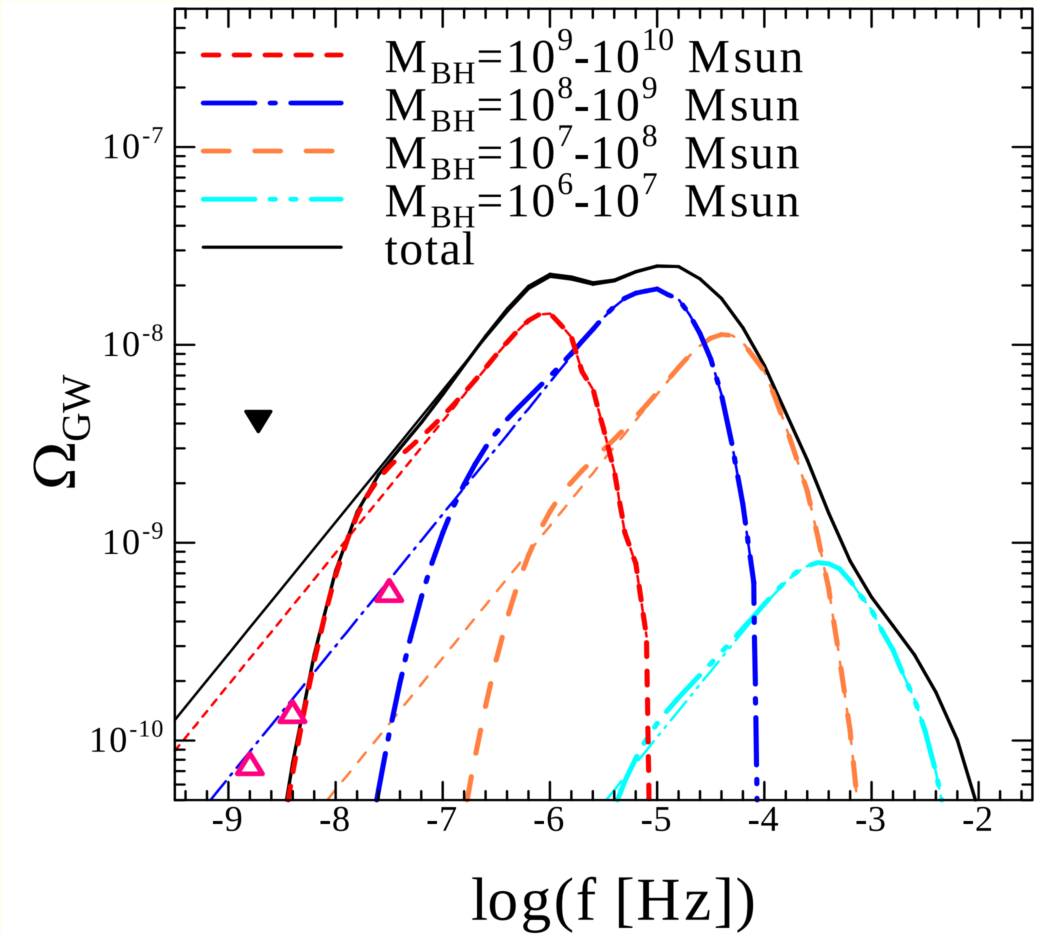


SNe feedback strength:



$e_0 = 0.8$ and $f_{p,0}/f_{p,\max} = 10^{-3}$.

*GWBG energy density from SMBH binaries



▼ : the current limit from pulsar timing measurements (Lommen 2002).

Thin lines: for e_0=0 (circular orbits).

Thick lines: for e_0 =0.8, $f_{p,0}/f_{max}=1/10^3$

power spectrum => energy density

$$\Omega_{GW}(f) = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f)$$

△ : the potential future lower limits from the full PPTA data-set for the case of $h_c \propto f^{-2/3}$ [e_0=0: circular orbits].

(Jenet et al. 2006)

The full PPTA data-set would be able to constrain the effect of eccentricity on the GWBR from SMBH binaries.

$f_{max} \propto 1/M_{BH}$

§ 5.Summary

- We have formulated the power spectrum of GWBR from cosmological compact binaries ***on eccentric orbits***.
- Then using the formulation and our SA-model for galaxy + SMBH formation, we have calculated power spectra of GWBR from coalescing SMBH binaries ***on eccentric orbits.***

=> Resultant power spectra of the GWBR from SMBH binaries ***on eccentric orbits*** are suppressed owing to the harmonics radiation for lower frequencies $f < 1$ nHz if the initial eccentricity is $e_0 > 0.2$ at a = 300 R_S.

- The research about the initial eccentricity distribution is very important and essential.
- The overall shape and amplitude of the power spectrum of the GWBR depend on galaxy formation processes.
- The pulsar timing observations such as PPTA project would be able to constrain not only the number density of SMBH binaries but also ***the effect of orbital eccentricity.***